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there perhaps a better way to phrase the control question? And once this question is put, the whole matter devolves once again into asking what the vocabulary is to be used for and how the vocabulary is to be defined. To affirm that this control question is *the* one in which we are interested seems hardly enough.

The best example of why this control question, rigorously and narrowly and systematically employed, may lead to erroneous results is that of the uncle and aunt mentioned above. With the possible exception of lunatics, morons and humorists, almost every adult American who is asked the control question about uncle's wife and aunt's husband will reply that "she is may aunt" and "he is my uncle" and these meanings then are added to the denotata. But when the cousin's spouse, the spouse of the nephew or niece or the spouse's nephew or niece are asked about in this way, discrepant, variable material is elicited: the material will show a high degree of variance from informant to informant and from the same informant at different times.

With this rigorous eliciting procedure, limited to the systematic use of this control question, there is no way to discover that the English word "relationship" has two meanings, not just one, and that where the field worker thinks he is asking about the classification in kinship terms of a kind of person he may be eliciting information on how the interpersonal relationship is known, and on the fact that "aunt" and "uncle" are appropriate terms of respect from junior to senior as well as terms which mark a kind of interpersonal relationship.

Once it is clear that more is needed than just a rigorously applied control question we are back where we were at the very outset. Are we after linguistic material for linguistic purposes? Are we after the cultural definitions, content, character, etc. of kin categories? The kind of a control question we use depends very much not only on what kind of data we seek, but also the way in which those data occur in the culture with which we are dealing.

Whether a dictionary of kinship terms is finite or not is thus a question that can only be closed after we know the aims of the dictionary, the uses to which it is to be put, and something about the definition of the dictionary itself, and these depend in part at least on the culture from which it is abstracted.

1. Schneider, D.M., *American Kinship: A Cultural Account.* Prentice-Hall. 1968.
2. Schneider, D.M. and Homans, G.C, "Kinship Terminology and the American Kinship System." *AA*, 57, 1955.
3. Whether Goodenough's distinction between basic and derivative terms (a derivative term is a basic term + a modifier) will help to clarify this problem or not remains to be seen, but it does permit the question of cousins to be dealt with with considerable economy. This is accomplished by treating "cousin" as the basic term, "first, second, etc." and "removal" as modifiers, thus permitting the infinite series to be described in just that way.
4. McLennan, J.F., *Studies in Ancient History.* 1886. pp. 249–301.
5. Schneider, D.M. and Homans, G.C., *op. cit.*
6. Goodenough. W.H., "Yankee Kinship Terminology: a problem in componential analysis." in Hammel, E.A. (e.d.), Formal Semantic Analysis., *AA*, 67, 1965. p. 262.

Formal Analysis of Prescriptive Marriage System: The Murngin Case

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In 1949 André Weil proposed a unique concept of 'marriage types' for the analysis of section systems. He wished to prove that the theory of groups of permutations is applicable to the study of prescribed marriage systems. Weil's idea was developed by a group of mathematicians, though sporadically, and thus a kind of new mathematical approach to the kinship structure has been established. Following Weil, Robert Bush in his undated mimeo manuscript introduced the 'permutation matrices' as a more effective tool for analysis. Kemeny, Snell and Thompson (1956) first systematized the properties of the prescriptive marriage systems as an integrated set of axioms. All distinct kinship structures which satisfy the above-mentioned axioms are systematically derived and described by Harrison White (1963), who adopts more practical 'generators' to set the structural analysis on a more concrete bases.

In the past two decades the kinship mathematics developed by the mathematicians has made a remarkable progress, but there still remain controversial problems, and the effective range of the mathematical approach is obviously limited. White himself recognized the failure of his structural analysis of those societies such as Murngin or Purums, which practice unilateral cross-cousin marriage. There may exist many reasons for this failure, as discussed by White and Reid (1967), but it is time to question whether the method they applied is sound or not.

If we examine the writings of Weil and his successors, the fatal fact emerges that their method does not allow distinction of the oblique marriage from the cross-cousin marriage. This defect is caused by the fact that the mathematicians neglect important anthropological phenomena, as for example, the regulation of marriage rules among the descent groups.

Here I propose the following working hypotheses for a new mathematical approach to the prescriptive marriage systems.

(1) The matrilateral cross-cousin marriage system is described by an $\underline{n}$-generation cycle or circulation, where the marriage cycle (or circulation connubium) is derived from $\underline{n}$ hordes or clans, the minimum number for $\underline{n}$ being 3.

(2) Under the same condition the patrilateral cross-cousin marriage system is described by a 2-generation cycle, or the so-called cycle of 'alternating generations.'

(3) The bilateral cross-cousin marriage system is described by the above-mentioned two principles, the '$\underline{n}$-generation' and '2-generation' cycles, but in this case $\underline{n}$ may be 2 or more.

The present study is designed to examine only the first hypothesis. The Murngin case is chosen. Methodologically, the new mathematical approach proposed by Harvey and Liu (1967) is adopted. There all of the kin relationships are reduced to the two basic units of 'parent' and 'child'. 'Kinship category' is determined by the products of the two units as generators. The kinship categories represented by the numerical notation system are computable, and mathematically they are regulated by the algebraic theory.

The model of genealogical space composed by the societies of the section system is completely different from that composed by the societies characterized by 'kinship categories' or 'bilateral systems.' Therefore, the numerical kinship notation system is not applicable to them. New generators appropriate to the section system must be selected. For this purpose I propose here two basic units, 'father-child link' and 'mother-child link.' The former is represented by $\underline{m}$ and the latter by $\underline{f}$, which are used as generators for the analysis of the Murngin system.

It is an established theory that at least three descent groups are necessary to form a marriage cycle for the practice of the so-called 'remote exchanges' characterized by the unilateral cross-cousin marriages. The Murngin's sixty-odd patrilineal clan-hordes, which are the minimum unit of an exogamous group, are divided into two moieties which intermarry. These are known as *Dua* and *Yiritcha* respectively. Each moiety is further subdivided into four sections, making a total of eight sections. For *Dua* the sections are: *Buralang* (O), *Warmut* (P), *Balang* (Q), and *Karmarung* (R). For *Yiritcha* they are: *Bulain* (S), *Kaijark* (T), *Ngarit* (U), and *Bangardi* (V). (In parenthesis are given the symbols used in this paper for the respective sections.)

According to the Murngin law a marriage among the clans of the same moiety is prohibited, so the number of clans or hordes required to organize the marriage cycle must be even, and the minimum number is four. Figure 1 depicts the marriage cycle of the matrilateral cross-cousin marriage system composed by four clans which are represented by a patri-line. The alphabetical order represents four patri-clans and the numbers four matri-clans. Solid lines stand for patri-lines and dotted lines for matri-lines.

The diagram shows the hypothesis (1) working: each descent-line is regulated by a 4-generation cycle, thus the four patri-lines and the four matri-lines are intersecting each other to produce sixteen segments. In Figure 2 the general symbols used in Figure 1 are replaced by the specific section symbols that are used for the Murngin system. Clans belonging to the same moiety is distinguished by the subscripts.

The Murngin system is further characterized by the extraordinary rule of pairing two marriage classes, regular and alternate, for each section. These characteristics are best shown in the Cayley diagram (Fig. 3) where each segment is represented by the accumulation of generators m and f. Both generators are regulated in modulo 4, and O_1 is chosen as origin or for the position of ego (I). Arrows on the matrilineal path indicate the direction of transit from ascending to descending generation. The lines of descent are constant and not affected by the type of marriage. The patrilineal path is reversible. In the case of regular marriage, it proceeds in the direction of alphabetical order, and for alternate marriage in reverse order.

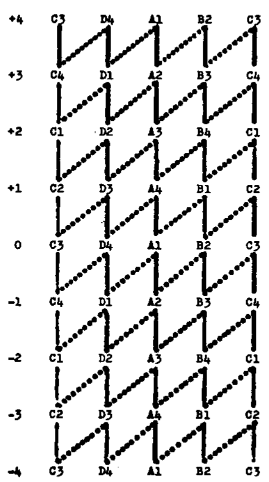

Fig. 1

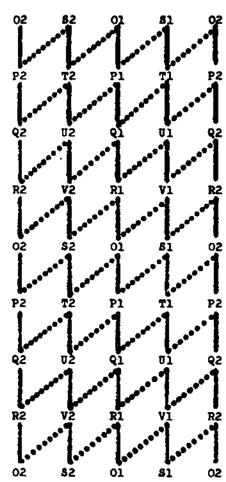

Fig. 2

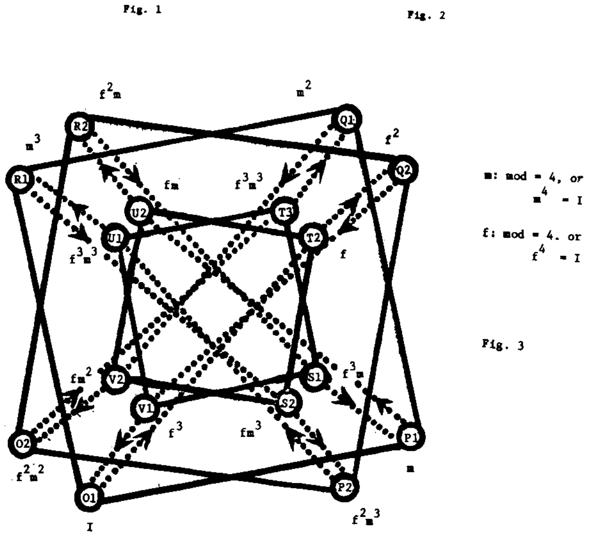

Fig. 3

Each segment plays multiple roles and expresses multiple conbinations of generators. For example

$$R_2 = mf^2, f^2m, fmf, mf^{-2}, f^{-2}m, f^{-1}mf^{-1}, m^{-3}f^2, m^{-3}f^{-2},..$$

Among them the form f^nm^n is chosen as a representative for each segment. To express this a new numerical notation system is given. We adopt a two place system and assign the first place to the exponent of f and the second place to the exponent of m. Thus, for example, f^2m (R_2) is represented as 21. Now the Murngin's basic marriage cycle, composed of four clan-hordes (G_4^4: superscripts indicate the generation cycle and subscripts indicate the number of hordes), can be expressed as a set of 16 segments in the following diagram.

$$G_4^4 = \left\{ \begin{array}{llll} 00\,(O_1) & 10\,(T_2) & 20\,(Q_2) & 30\,(V_1) \\ 01\,(P_1) & 11\,(U_2) & 21\,(R_2) & 31\,(S_1) \\ 02\,(Q_1) & 12\,(V_2) & 22\,(O_2) & 32\,(T_1) \\ 03\,(R_1) & 13\,(S_2) & 23\,(P_2) & 33\,(U_1) \end{array} \right\}$$

$$m\text{: mod} = 4$$

$$f\text{: mod} = 4$$

This segment diagram shows an algebraic group characterized by the following mathematical properties.

(1) Binary operation: The generations of the same descent line (or the exponents of the same generators) are computable. The formula is $f^pm^q \cdot f^rm^s = f^{p+r}m^{q+s}$.

For example, $23 \cdot 31 = 10$ (mod, m, f = 4).

(2) Identity: 00 is the identity segment. For example, $00 \cdot 21 = 21 \cdot 00 = 21$.

(3) Associativity: This property is satisfied as $(21 \cdot 13) \cdot 02 = 21 \cdot (13 \cdot 02) = 32$.

(4) Inverses: Each element or segment has its inverse in the set. The formula is $(f^pm^q)^{-1} = f^{4-p}m^{4-q}$. For example, $(21)^{-1} = 23$, or $21 . 23 = 00$.

(5) Commutative: This property is shown as $13 . 22 = 22 \cdot 13 = 31$.

Because it is commutative, and because of the finite number of its elements, this group is called a 'finite Abelian group.' In the segment diagram each column represents a patrilineal sub-group. Each row represents a matri-lineal sub-group. The multiplication table of this group is

·	00	01	02	03	10	11	12	13	20	21	22	23	30	31	32	33
00	00	01	02	03	10	11	12	13	20	21	22	23	30	31	32	33
01	01	02	03	00	11	12	13	10	21	22	23	20	31	32	33	30
02	02	03	00	01	12	13	10	11	22	23	20	21	32	33	30	31
03	03	00	01	02	13	10	11	12	23	20	21	22	33	30	31	32
10	10	11	12	13	20	21	22	23	30	31	32	33	00	01	02	03
11	11	12	13	10	21	22	23	20	31	32	33	30	01	02	03	00
12	12	13	10	11	22	23	20	21	32	33	30	31	02	03	00	01
13	13	10	11	12	23	20	21	22	33	30	31	32	03	00	01	02
20	20	21	22	23	30	31	32	33	00	01	02	03	10	11	12	13
21	21	22	23	20	31	32	33	30	01	02	03	00	11	12	13	10
22	22	23	20	21	32	33	30	31	02	03	00	01	12	13	10	11
23	23	20	21	22	33	30	31	32	03	00	01	02	13	10	11	12
30	30	31	32	33	00	01	02	03	10	11	12	13	20	21	22	23
31	31	32	33	30	01	02	03	00	11	12	13	10	21	22	23	20
32	32	33	30	31	02	03	00	01	12	13	10	11	22	23	20	21
33	33	30	31	32	03	00	01	02	13	10	11	12	23	20	21	22

Next, let us expand the marriage cycles to have six, eight or more clans participating. According to our working hypothesis (1), such systems should be regulated by a 6-generation cycle and produce 6^2 segments in the first case, or regulated by an 8-generation cycle and produce 8^2 segments in the second case. However, in the Murngin system the 4-generation cycle of each horde is fixed and invariable regardless of the number of hordes participating in the marriage cycle. This is not fully consistent with our working hypothesis, which states that the generation cycle should be equal to the number of clans or hordes. The hypothesis must revised to make it consistent with the properties of Murngin system.

Figures 4 and 5 show diagrams of matrilateral cross-cousin marriage organized from six or eight patri-clans or hordes. They produce 4x6 and 4x8 segments respectively. The generation cycles of the matri-lines are not four. Now they are determined by the least common multiple (LCM) of the generation cycle of patrilines and the number of patrilines. Then the number of the matrilines is determined by the quotient of the total number of segments and the generation cycle of the matri-line. Thus in the six patri-line case, the matri-line is two and regulated by a 12-generation cycle; in the eight patri-line case, the matri-line is four and regulated by a 8-generation cycle. These groups are expressible in the following segment diagrams respectively. (1)

The general form of the Murngin marriage cycle can be given in a segment diagram. (2)

The compound groups of circulation connubium or marriage cycle of the Murngin system (G) can be expressed in the following canonical form:

$$G = G_4^4 + G_6^4 + G_8^4 + \cdots\cdots + G_{2n}^4.$$

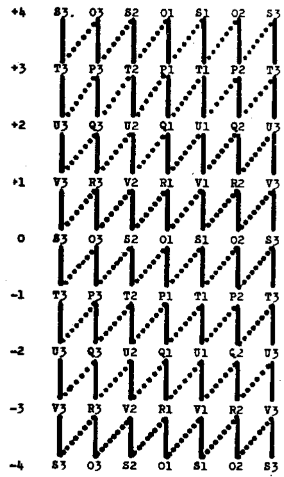

Fig. 4

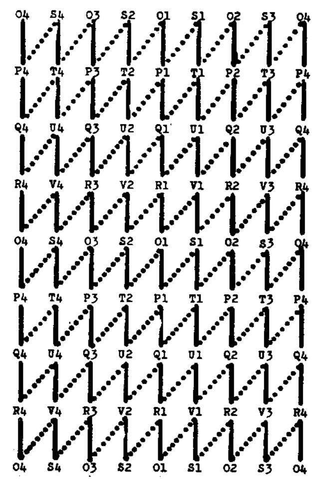

Fig. 5

(1)

$G_6^4 =$

00 (O_1)	10 (T_2)	20 (Q_3)	30 (V_3)	40 (O_2)	50 (T_1)
01 (P_1)	11 (U_2)	21 (R_3)	31 (S_3)	41 (P_2)	51 (U_1)
02 (Q_1)	12 (V_2)	22 (O_3)	32 (T_3)	42 (Q_2)	52 (V_1)
03 (R_1)	13 (S_2)	23 (P_3)	33 (U_3)	43 (R_2)	53 (S_1)

m: mod = 4
f: mod = 12

$G_8^4 =$

00 (O_1)	10 (T_2)	20 (Q_3)	30 (V_4)	40 (O_4)	50 (T_3)	60 (Q_2)	70 (V_1)
01 (P_1)	11 (U_2)	21 (R_3)	31 (S_4)	41 (P_4)	51 (U_3)	61 (R_2)	71 (S_1)
02 (Q_1)	12 (V_2)	22 (O_3)	32 (T_4)	42 (Q_4)	52 (V_3)	62 (O_2)	72 (T_1)
03 (R_1)	13 (S_2)	23 (P_3)	33 (U_4)	43 (R_4)	53 (S_3)	63 (P_2)	73 (U_1)

m : mod = 4, f: mod = 8

(2) $G_{2n}^4 =$

00	10	20	–	–	–	(2n−2)0	(2n−1)0
01	11	21	–	–	–	(2n−2)1	(2n−1)1
02	12	22	–	–	–	(2n−2)2	(2n−1)2
03	13	23	–	–	–	(2n−2)3	(2n−1)3

m: mod = 4
f: mod = LCM of 4 and 2n

Conclusion

In this study a new numerical kinship notation system for section system is presented. We adopt two basic kinship units, 'father-child link' and 'mother-child link' as generators for the analysis of the Murngin system. The groups of circulation connubium or marriage cycle are demonstrated in a segment diagram, and their mathematical properties are characterized as the 'finite Abelian groups.' Meanwhile, the working hypothesis proposed for the analysis of matrilateral cross-cousin marriage system has been shown to be incomplete. It needs to be revised as follows, in order to make it consistent with the properties of the Murngin system.

The matrilateral cross-cousin marriage system is described by an $\underline{n}$-generation cycle or circulation, where the marriage cycle (or circulation connubium) is derived from $\underline{n}$ hordes or clans, the minimum number for $\underline{n}$ being 3. When the generation cycle governing the horde or clan is fixed as $\underline{p}$, and $\underline{p} \geq 1$, then the other descent-line (if the former is patrilineal, the latter is matrilineal, or vice versa) will be regulated by a $\underline{q}$-generation cycle, with $\underline{q}$ being the least common multiple of $\underline{p}$ and $\underline{n}$.

References Cited

Bush, Robert R., 1963 – "An algebraic treatment of rules of marriage and descent." Appendix II of *An anatomy of kinship*, H. C. White. Englewood Cliffs, N.J., Prentice-Hall. pp. 159–172.

Haevey, John H. T. and Pin-hsiung Liu, 1967 – Numerical kinship notation system: mathematical model of genealogical space. *Bulletin of Institute of Ethnology*, Academia Sinica 23:1–22.

Kemeny, J. G., Snell, J. L. and G. L. Thompson, 1956 – *Introduction to finite mathematics*. Englewood Cliffs, N. J., Prentice-Hall. pp. 343–353.

Reid, Russell M., 1967 –"Marriage systems and algebraic group theory: a critique of White's An anatomy of kinship," *American Anthropologist*, 69, 171–178.

Weil, André, 1949 – Sur l'étude algebrique de certain types de lois de marriage (système Murngin). In Appendice à la Première Partie, *Les structures élémentaire de la parenté*, Claude Lévi-Strauss, Paris, Presses Universitaires de France, pp. 278–285. Also translated by C. White as Appendix I in *An anatomy of kinship*, H. C. White. Englewood Cliffs, N. J., Prentice-Hall.

White, Harrison C., 1963 – *An anatomy of kinship: mathematical models for structures of cumulated roles.* Englewood Cliffs, N. J., Prentice-Hall.